Eur. Phys. J. B **21**, 61–65 (2001)

The European Physical Journal B

EDP Sciences

Impurity scattering in f-wave superconductor UPt_3

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Received 11 October 2000

Abstract. We study theoretically the effect of impurity scattering in f-wave (or E_{2u}) superconductors. The quasi-particle density of states of f-wave superconductor is very similar to the one for d-wave superconductor as in hole-doped high T_c cuprates. Also in spite of anisotropy in $\Delta(\hat{\mathbf{k}})$, both the reduced superfluid density and the reduced electronic thermal conductivity is completely isotropic.

PACS. 74.20.-z Theories and models of superconducting state – 74.25.Bt Thermodynamic properties – 74.25.Fy Transport properties (electric and thermal conductivity, thermoelectric effects, etc.)

1 Introduction

After a long controversy, f-wave (or E_{2u}) superconductivity in UPt_3 is established in 1996 [1]. First the thermal conductivities with the heat current parallel to the c axis and within the basal plane decrease linearly in T the temperature at low temperature [2]. This is consistent with E_{2u} but inconsistent with the then prevailing model E_{1g} [3,4]. Almost at the same time ^{195}Pt Knight shift measurement in UPt_3 finds the spin triplet pair [5]. Later they find also among three phases A, B and C in UPt_3, only B phase is nonunitary [5]. This is again consistent with E_{2u} but not with E_{1g}. The details of the spin configuration of different phases are described in [6]. In the limit $\mathbf{H}$ goes to zero the order parameter in the B phase reverts to the doubly degenerate unitary one.

So we can write down the superconducting order parameter

$$\Delta(\hat{\mathbf{k}}) = \alpha\Delta\hat{\mathbf{d}}k_3(k_1 \pm \mathrm{i}k_2)^2 \tag{1}$$

and $\alpha = \frac{3\sqrt{3}}{2}$, $\hat{\mathbf{d}} \parallel \mathbf{c}$ and $\mathbf{k}$ is the quasi-particle momentum.

Very recently we have shown that f-wave superconductivity describes quite well the observed upper critical field in UPt_3 [7,8].

The object of this paper is to study the impurity effect in f-wave superconductivity. It is well known that impurity produces profound effect in unconventional superconductors [9,10]. Also for the analysis of transport properties the impurity scattering is crucial. Following [3,4] we assume that the impurity scattering is in the unitarity limit. Then we analyze the thermodynamic and transport properties. In particular we find both the reduced superfluid density and the reduced electronic thermal conductivity are completely isotropic (or $\rho_s(T)/\rho_s(0)$ and $\kappa_{el}(T)/\kappa_{el}(0)$ are isotropic).

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The present result describes reasonable well the temperature dependence of the ultrasonic attenuation coefficient of UPt_3 [11,12]. Also the deviation from the universal limit in the low temperature thermal conductivity [13] in f-wave superconductor is very similar but somewhat larger than in d-wave superconductor [9], which may be consistent with the recent measurement of electron-irradiated UPt_3 [14]. A part of the present work has been reported in [15]

2 Formulation

Following the standard approach the effect of impurity scattering is incorporated by replacing the frequency ω in the quasi-particle Green function in the Nambu space by the renormalized one $\tilde{\omega}$.

$$G^{-1}(\omega, \mathbf{p}) = \tilde{\omega} - \xi\rho_3 - \alpha\Delta\rho_1 k_3(k_1 \pm \mathrm{i}k_2\rho_3)^2\sigma_1 \tag{2}$$

and

$$\tilde{\omega} = \omega + \mathrm{i}\Gamma\left\langle \frac{\tilde{\omega}}{\sqrt{\tilde{\omega}^2 - \Delta^2 f^2}} \right\rangle^{-1} \tag{3}$$

where ρ_i are the Pauli matrices in the Nambu space, $\Gamma = n_i/\pi N_0$ the scattering rate, $f = \alpha \sin^2\theta\cos\theta$ and $\langle\cdots\rangle$ means the average on the Fermi surface.

Then the Gap equation is given by

$$\lambda^{-1} = 2\pi T \frac{1}{\langle |f|^2\rangle} {\sum_n}' \left\langle \frac{|f|^2}{\sqrt{\tilde{\omega}_n^2 + \Delta^2|f|^2}} \right\rangle \tag{4}$$

here $\langle f^2\rangle = \frac{18}{35}$, λ is the dimensionless coupling constant, and $\tilde{\omega_n}$ is the renormalized Matsubara frequency. When

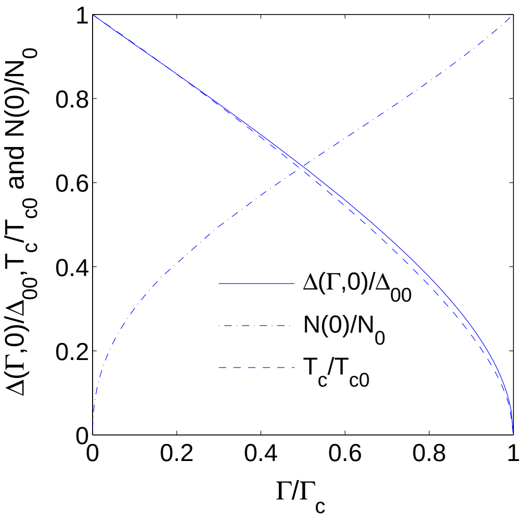

Fig. 1. $T/T_{c0}, \Delta(\Gamma,0)/\Delta_{00}$ and $N(0)/N_0$ *versus* Γ/Γ_c.

$\Delta \to 0$, we obtain the universal Abrikosov-Gor'kov formula [16]

$$-\ln\left(\frac{T_c}{T_{c0}}\right) = \psi\left(\frac{1}{2}+\frac{\Gamma}{2\pi\Gamma_c}\right) - \psi\left(\frac{1}{2}\right) \tag{5}$$

where $T_c(T_{c0})$ is the superconducting transition temperature in the presence (absence) of impurities. Also we have $T_c = 0$ when $\Gamma = \Gamma_c = \frac{\pi}{2\gamma}T_{c0} = 0.8819T_{c0}$. For $T = 0K$, equation (4) reduced to

$$\begin{aligned}-\ln\left(\frac{\Delta(\Gamma,0)}{\Delta_{00}}\right) = \frac{1}{\langle f^2\rangle}\Bigg\{&-C_0\left\langle\frac{f^2}{\sqrt{C_0^2+f^2}}\right\rangle \\ &+\left\langle f^2\ln\left(\frac{C_0+\sqrt{C_0^2+f^2}}{f}\right)\right\rangle \\ &+\zeta\int_{C_0}^{\infty} \mathrm{d}u\langle\frac{f^2}{(u^2+f^2)^{3/2}}\rangle\langle\frac{1}{\sqrt{u^2+f^2}}\rangle^{-1}\Bigg\}\end{aligned} \tag{6}$$

where $\zeta = \frac{\Gamma}{\Delta}$ and $\mathrm{i}C_0 = u(\equiv \frac{\tilde{\omega}}{\Delta})|_{\omega=0}$ is given by

$$C_0^2 = \zeta\langle\frac{1}{\sqrt{C_0^2+f^2}}\rangle^{-1} \Rightarrow \sqrt{3}\zeta\left[\ln\left(\frac{1}{C_0}\right)+\text{const.}\right]^{-1} \tag{7}$$

where the last expression is the limiting value for $\zeta \to 0$.

Also the residual density of states is given by

$$\frac{N(0)}{N_0} = C_0\langle\frac{1}{\sqrt{C_0^2+f^2}}\rangle = \frac{\zeta}{C_0} = \frac{\Gamma}{\Delta C_0}\,\cdot \tag{8}$$

We show in Figure 1 $T_c/T_{c0}, \Delta(\Gamma,0)/\Delta_{00}$, and $N(0)/N_0$ *versus* Γ/Γ_c. This figure is strikingly similar to the one in d-wave superconductor [9].

The quasi-particle density of states is given by

$$\frac{N(E)}{N_0} = \mathrm{Re}\left\langle\frac{\tilde{\omega}}{\sqrt{\tilde{\omega}^2-\Delta^2 f^2}}\right\rangle \tag{9}$$

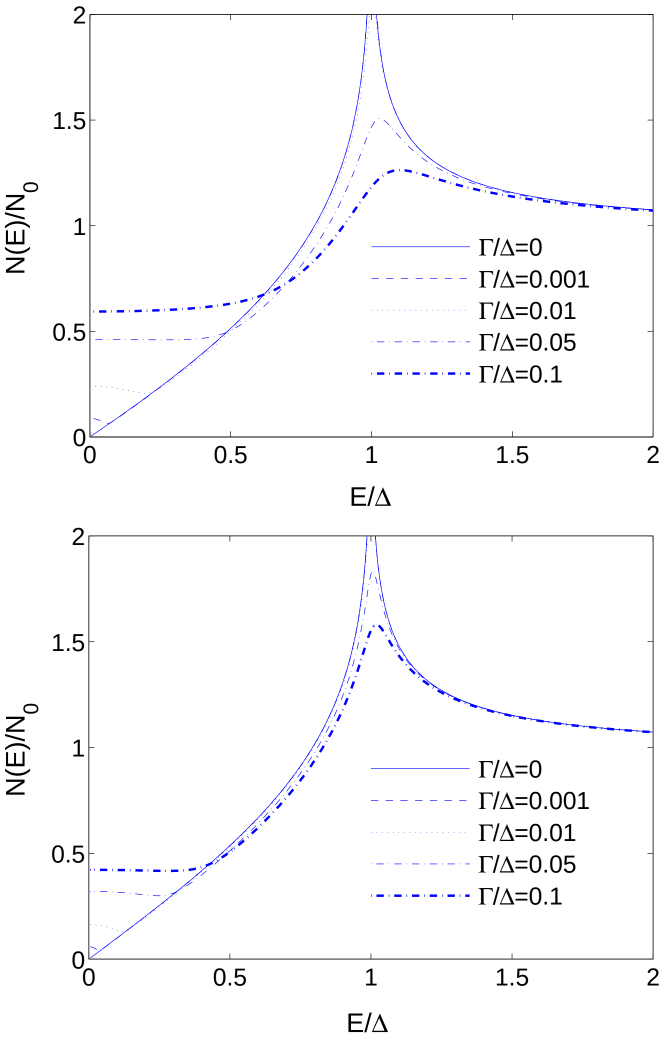

Fig. 2. The quasi-particle density states $N(E)/N_0$ *versus* E/Δ for several Γ/Δ. a) for f-wave, b) for d-wave superconductor respectively.

where we put $E = \omega$ in equation (3). This is shown for a few $\zeta = \Gamma/\Delta$ *versus* E/Δ in Figure 2a. We display in Figure 2b the corresponding one for d-wave superconductor [17]. They are also very similar except for large value of ζ. For larger ζ, $N(E)/N_0$ for f-wave superconductor approaches the normal state result much faster.

Also equation (4) is solved numerically and we show $\frac{\Delta(\Gamma,T)}{\Delta_{00}}$ for a few $g = \Gamma/\Gamma_c$ *versus* T/T_{c0} in Figure 3.

3 Thermodynamics

It is convenient to start with the entropy given by

$$\begin{aligned}S &= -4\int_0^{\infty} \mathrm{d}EN(E)\left[f\ln f + (1-f)\ln(1-f)\right] \\ &= 4\int_0^{\infty} \mathrm{d}EN(E)\left[\beta E(1+\mathrm{e}^{\beta E})^{-1} + \ln(1+\mathrm{e}^{-\beta E})\right].\end{aligned} \tag{10}$$

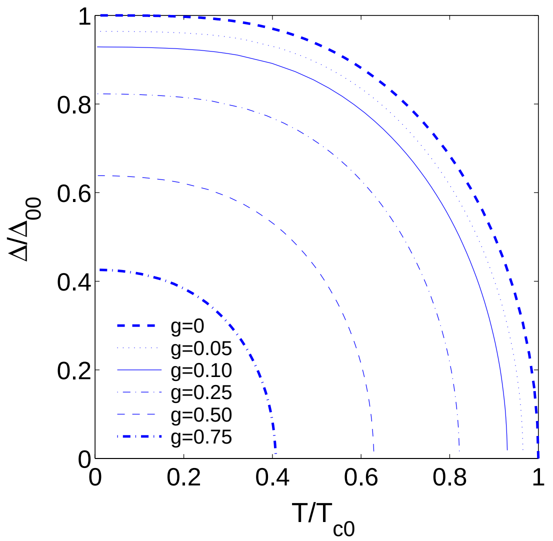

Fig. 3. $\Delta(\Gamma,T)/\Delta_{00}$ *versus* T/T_c for several $g = \Gamma/\Gamma_c$.

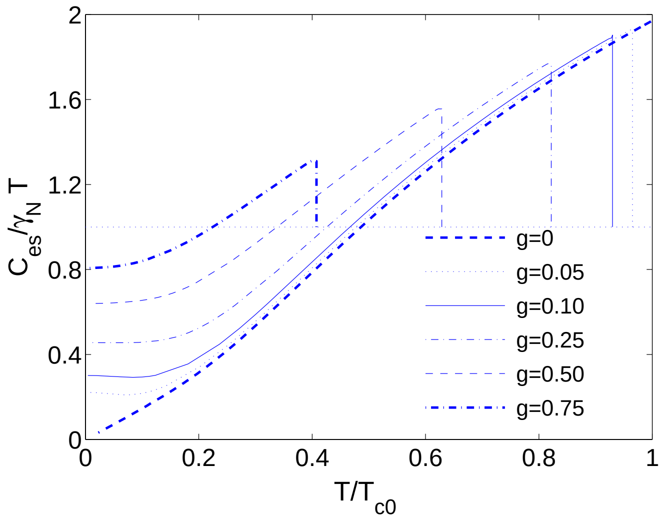

Fig. 5. $C_s/\gamma_N T$ *versus* T/T_{c0}.

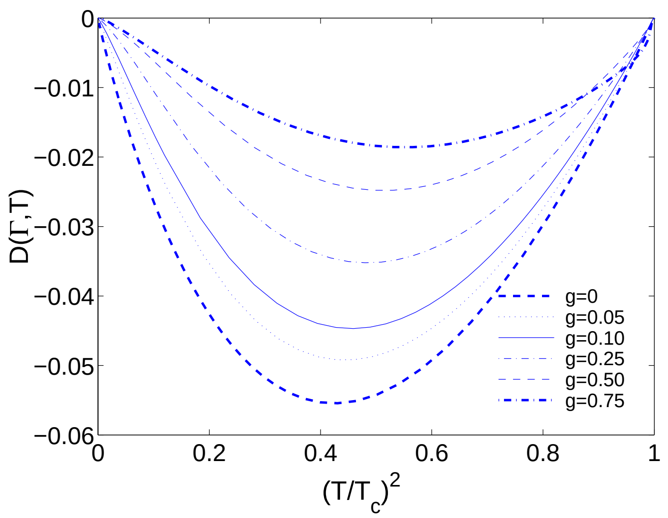

Fig. 4. $D(T/T_c)$ *versus* $(T/T_c)^2$. In the pure limit $|D(T/T_c)|$ is much larger than the one for s-wave superconductor. But $|D(T/T_c)|$ decrease monotonically with increasing $g = \Gamma/\Gamma_c$.

Then we obtain

$$\frac{H_c^2(T)}{8\pi} = \int_T^{T_c} \mathrm{d}T(S_n(T) - S(T)) \tag{11}$$

and $S_n(T) = \frac{2\pi^2}{3}N_0 T$ the entropy in the normal state and $H_c(T)$ is the thermodynamic critical field. In Figure 4 we show $D(\frac{T}{T_c})$ *versus* $(T/T_c)^2$, where

$$D\left(\frac{T}{T_c}\right) = \frac{H_c(T)}{H_c(0)} - \left(1 - \left(\frac{T}{T_c}\right)^2\right) \tag{12}$$

for several $g = \Gamma/\Gamma_c$.

Also the specific heat is given by $C_s = T\frac{\mathrm{d}S}{\mathrm{d}T}$. In Figure 5 we show $C_s/\gamma_n T$ where $\gamma_n = \frac{2\pi^2}{3}N_0$. The specific heat thus obtained is quite consistent with the observation [18].

We are concerned here with the phase T_{c-}. Perhaps the more details of the theory can be tested within the system with controlled impurity concentration [14].

Finally the reduced superfluid density is isotropic and given by

$$\rho_s(\Gamma,T) = 2\pi T\sum_{n=0}^{\infty}\left\langle \frac{\Delta^2 f^2}{(\tilde{\omega}_n^2 + \Delta^2 f^2)^{3/2}}\right\rangle. \tag{13}$$

Here we put $\rho_s(0,0) = 1$. This follows from the relation

$$3\int_0^1 z^2 F(f) = \frac{3}{2}\int_0^1 (1-z^2)F(f) = \int_0^1 \mathrm{d}zF(f) \tag{14}$$

where F is an arbitrary function of $f = \frac{3\sqrt{3}}{2}z(1-z^2)$.

First at $T = 0$ K, equation (13) reduces to

$$\rho_s(\Gamma,0) = 1 - \frac{N(0)}{N_0} + \zeta\int_{C_0}^{\infty}\mathrm{d}u\left(\frac{\left\langle\frac{f^2}{(u^2+f^2)^{3/2}}\right\rangle}{\left\langle\frac{u}{\sqrt{u^2+f^2}}\right\rangle}\right)^2. \tag{15}$$

This is shown in Figure 6 *versus* Γ/Γ_c together with the one in d-wave superconductor. They are almost indistinguishable one from the other.

Finally $\rho_s(\Gamma,T)$ for several g *versus* $\frac{T}{T_c}$ is shown in Figure 7. In the pure limit $\rho_s(\Gamma,T)$ decrease linearly with T as in d-wave superconductor [9,19].

4 Transport properties

Following [21] the ultrasonic attenuation coefficient for the transverse wave with $\mathbf{q} \parallel \mathbf{b}$ and $\mathbf{e} \parallel \mathbf{c}$ and $\mathbf{q} \parallel \mathbf{b}$ and $\mathbf{e} \parallel \mathbf{a}$

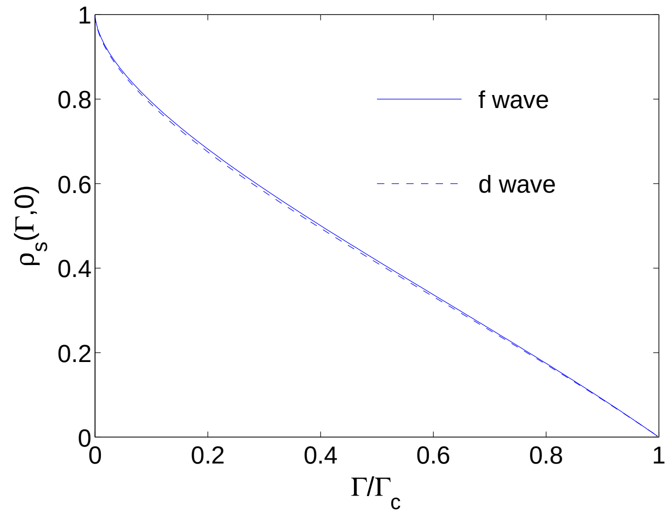

Fig. 6. $\rho_s(\Gamma, 0)$ *versus* Γ/Γ_c. This behavior is very similar to the one in d-wave superconductor.

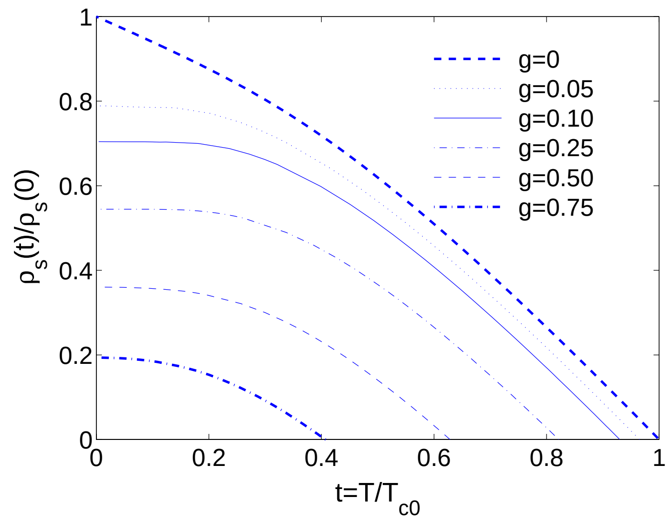

Fig. 7. $\rho_s(\Gamma, T)$ *versus* T/T_{c0} for several g.

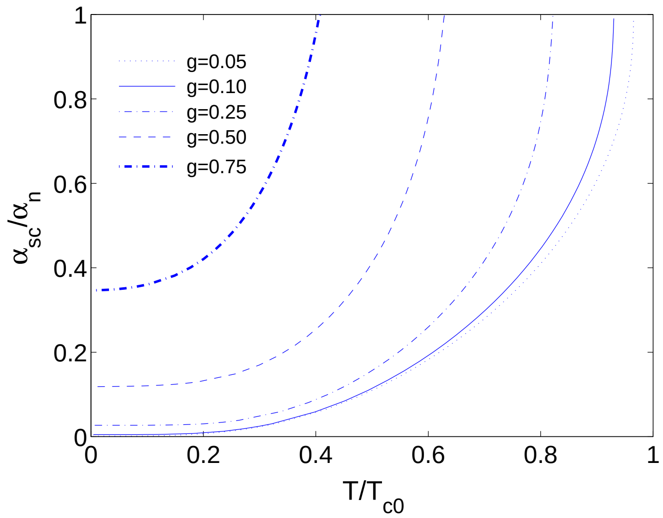

Fig. 8. α_{sc}/α_N *versus* T/T_{c0} for several g for $\mathbf{q} \parallel \mathbf{b}$ and $\mathbf{e} \parallel \mathbf{c}$.

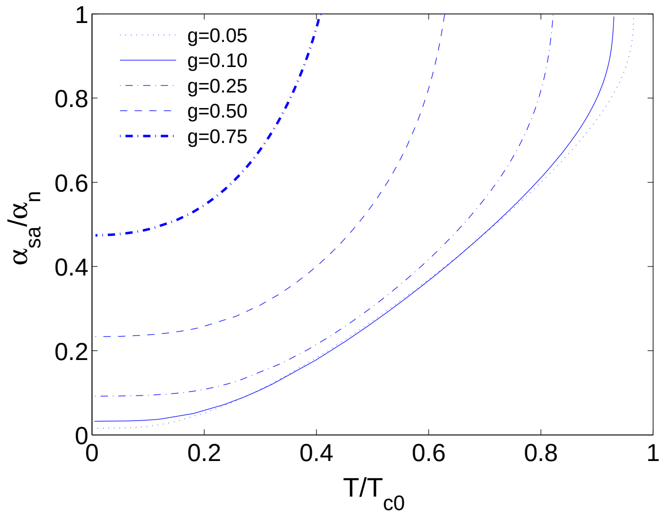

Fig. 9. α_{sa}/α_N *versus* T/T_{c0} for several g for $\mathbf{q} \parallel \mathbf{b}$ and $\mathbf{e} \parallel \mathbf{a}$.

are given by

$$\frac{\alpha_{sc}}{\alpha_N} = \frac{15\zeta}{2} \int_0^\infty \frac{\mathrm{d}E}{2T} \mathrm{sech}^2\left(\frac{E}{2T}\right) \times \int_0^1 \mathrm{d}z z^2(1-z^2) \frac{h(u,f)}{\mathrm{Im}\sqrt{u^2-f^2}} \tag{16}$$

and

$$\frac{\alpha_{sa}}{\alpha_N} = \frac{15\zeta}{8} \int_0^\infty \frac{\mathrm{d}E}{2T} \mathrm{sech}^2\left(\frac{E}{2T}\right) \times \int_0^1 \mathrm{d}z (1-z^2)^2 \frac{h(u,f)}{\mathrm{Im}\sqrt{u^2-f^2}} \tag{17}$$

respectively, where

$$h(u,f) = \frac{1}{2}\left(1 + \frac{|u|^2 - f^2}{|u^2 - f^2|}\right) \tag{18}$$

and $u = \tilde{\omega}/\Delta$ and $\omega = E$ in equation (3).

Equations (16) and (17) are evaluated for several g and shown in Figures 8 and 9 respectively. Recently the ultrasonic attenuation in f-wave superconductor is obtained in the clean limit [20]. The present result is consistent with both the experiment and the clean limit result. But it appears to predict somewhat steeper decrease near $T = T_c$ in the attenuation coefficient than observed experimentally [11,12].

Finally the thermal conductivity is given by

$$\frac{\kappa_s(T)}{\kappa_n(T)} = \frac{3\Gamma}{2\pi^2\Delta} \int_0^\infty \frac{\mathrm{d}E}{2T^3} E^2 \mathrm{sech}^2\left(\frac{E}{2T}\right) \frac{h(u,f)}{\mathrm{Im}\sqrt{u^2-f^2}} \tag{19}$$

which is shown in Figure 10. Perhaps of more interest is

$$\frac{\kappa}{\kappa_0} = \lim_{T\to 0} \frac{\kappa_s(T)}{\kappa_{s0}(T)} = \frac{\sqrt{3}\Delta_{00}}{\Delta(\Gamma,0)} \left\langle \frac{C_0^2}{(C_0^2+f^2)^{3/2}} \right\rangle \tag{20}$$

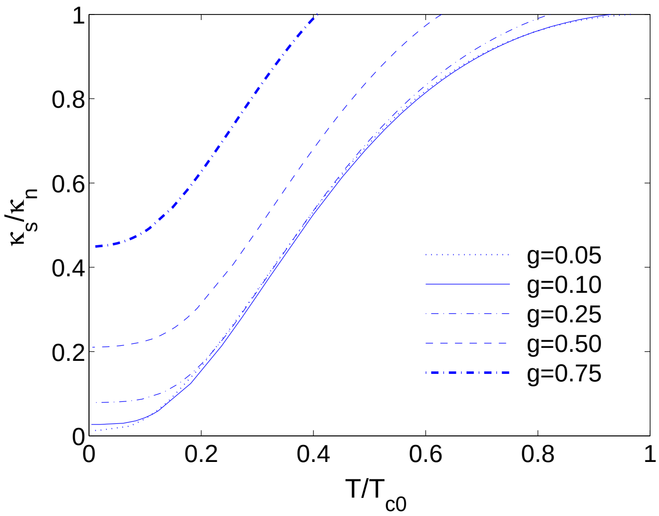

Fig. 10. $\kappa_s(T)/\kappa_n(T)$ *versus* T/T_{c0} for several g. Here $\kappa_n(T) = \frac{\pi^2 n}{3m\Gamma}T$, and n is the electron density.

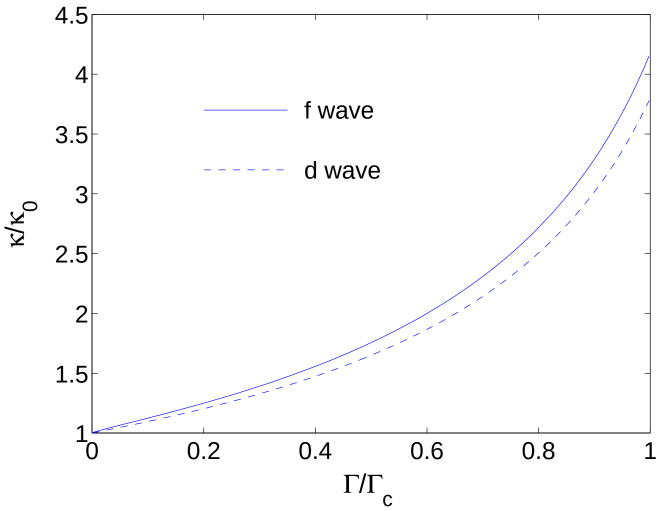

Fig. 11. $\kappa/\kappa_0 = \lim_{T\to 0}\kappa_s(T)/T\kappa_0$ *versus* Γ/Γ_c. Here $\kappa_0 = \lim_{\Gamma\to 0}\kappa_s(T)/T$, $\kappa/\kappa_0 > 1$ implies the deviation for the universality.

which is shown *versus* g in Figure 11. This describes the deviation from the universality ($\frac{\kappa}{\kappa(\Gamma,0)} = 1$ for $g \to 0$). We note the deviation is a little greater than the one for d-wave superconductors [9].

5 Summary

In summary we have studied the effect of impurity scattering in f-wave superconductors. We found f-wave superconductor behaves in many respects very similar to d-wave superconductor.

Also we predict the impurity dependence of the thermodynamic properties and transport properties of f-wave superconductors, which should be readily accessible to experiments. Finally the specific heat data from another triplet superconductor URu_2Si_2 [22] appears very consistent with f-wave superconductor.

We thank J.P. Brison, and J. Flouquet, T. Ishiguro and Y. Maeno for providing us the experimental data prior to publication. One of us (KM) thanks Max-Planck Institut für Physik Komplexer Systeme at Dresden for hospitality and continued support.

References

1. R.H. Heffner, M.R. Norman, Comments Cond. Matt. Phys. **17**, 361 (1996).
2. B. Lussier, B. Ellman, L. Taillefer, Phys. Rev. B **53**, 5145 (1996).
3. M.R. Norman, J.P. Hirschfeld, Phys. Rev. B **53**, 145 (1996).
4. M.J. Graf, S.-K. Yip, J.A. Sauls, J. Low Temp. Phys. **102**, 367 (1996).
5. H. Tou, Y. Kitaoka, K. Asayama, N. Kimura, Y. Onuki, E. Yamamoto, K. Maezawa, Phys. Rev. Lett. **77**, 1374 (1996); H. Tou, Y. Kitaoka, K. Ishida, K. Asayama, N. Kimura, Y. Juki, E. Yamamoto, Y. Haga, K. Maezawa, *ibid.* **80**, 3129 (1998).
6. Y. Kitaoka, H. Tou, K. Ishida, N. Kimura, Y. Ohnuki, E. Yamamoto, Y. Haga, K. Maezaw, Physica B **281-282**, 878 (2000).
7. G. Yang, K. Maki, Europhys. Lett. **48**, 208 (1999).
8. B.S. Shivaram, T.F. Rosenbaum, D.G. Hinks, Phys. Rev. Lett. **57**, 1256 (1986); K. Tenya, *et al.*, *ibid.* **77**, 3193 (1996).
9. Y. Sun, K. Maki, Phys. Rev. B **51**, 6059 (1995); Europhys. Lett. **32**, 355 (1995).
10. K. Maki, E. Puchkaryov, Europhys. Lett. **45**, 263 (1999); **50**, 533 (2000).
11. B.S. Shivaram, Y.H. Jeong, T.F. Rosenbaum, D.G. Hinks, Phys. Rev. Lett. **56**, 1078 (1986).
12. B. Elleman, L. Taillefer, M. Poirier, Phys. Rev. B **54**, 9043 (1996).
13. P.A. Lee, Phys. Rev. Lett. **71**, 1887 (1993).
14. J.P. Brison, H. Suderow, S. Kambe, A. Huxley, J. Flouquet, F. Rullier-Albenque, Physica B **281-282**, 872 (2000).
15. K. Maki, G. Yang, Fizika (Zagreb) **8**, 345 (1999); G. Yang, K. Maki, Physica C **341-348**, 1955 (2000).
16. A.A. Abrikosov, L.P. Gor'kov, Sov. Phys. JETP **12**, 243 (1961).
17. T. Hotta, J. Phys. Soc. Jpn **62**, 274 (1993).
18. R.A. Fisher, *et al.*, Phys. Rev. Lett. **62**, 1411 (1989).
19. H. Won, K. Maki, Phys. Rev. B **49**, 1397 (1994); K. Maki, H. Won, Annal Physik **5**, 320 (1996).
20. K. Machida, T. Ohmi, Physica B **281-282**, 981 (2000).
21. L.P. Kadanoff, I.I. Falko, Phys. Rev. **136**, A1170 (1964); V. Ambegaokar, A. Griffin, Phys. Rev. **137**, A1151 (1964).
22. K. Hasselbach, J.R. Kirtley, P. Lejay, Physica B **186-188**, 201 (1996).